

Kobayashi-Maskawa matrix moduli, decay constants and form factors determination from experimental data

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(Dated: October 14, 2010)

The aim of the paper is to propose another tool for phenomenological analyses of experimental data from superallowed nuclear and neutron β decays, and from leptonic and semileptonic decays, that allows the finding of the most probable numerical form of the CKM matrix, as well as the determination of decay constants, f_P , and of various form factors $f_+(q^2)$, by using another implementation of unitarity constraints. In particular this approach allows the determination of semileptonic form factors that is illustrated on the existing data from $D \rightarrow \pi l \nu$ and $D \rightarrow K l \nu$ decays.

PACS numbers: 12.15.-y, 12.15 Hh, 12.15 Ff

I. INTRODUCTION

Within the Standard Model (SM) the flavor physics is encoded by Kobayashi-Maskawa (KM) matrix, [1], supposed to be unitary, matrix that describes the quark flavor mixing through four independent parameters: three mixing angles, θ_{ij} , $ij = 12, 13, 23$, and one CP -violating phase, δ . By consequence the experimental determination of KM matrix entries is essential for the validation of the SM, and for detection of new physics beyond it.

However the determination of KM entries is not an easy problem because of two different causes. The first one is theoretical, namely the mixing angles are not invariant quantities, their numerical values depend on the original KM form, [1], or on the present day form, [2], which is not rephasing invariant, see [3]. These shortcomings are harmless if one follows Jarlskog's solution. Starting with her first papers on KM matrix, [4], she proposed the determination of the quark mixing matrix in terms of directly measurable quantities, and, in the same time, invariant quantities. In this context an invariant quantity is one whose numerical value does not depend on the KM matrix form, or of its rephasing invariance. Jarlskog provided two such invariants: the KM moduli, and the celebrated J invariant, [5]. Ten years later, other invariants, the angles of unitarity triangles, appeared on the scene [6]. After few years it was realized that all the measurable quantities of quark mixing matrix are expressible in terms of four independent KM matrix moduli, see [7] and [8]. In these papers it was also shown that areas of all unitarity triangles are equivalent, and numerically equal to half of J invariant, and in our opinion this equality also solved the J sign problem, by imposing $J > 0$. As a conclusion we can state that the numerical values for all measured invariant quantities should be the same irrespective of the physical processes where they are involved.

The second difficulty comes from the experimental side. If one wants the use of KM moduli as independent parameters these ones are not directly measured by experimenters. In the simplest case, that of leptonic decays, they measure branching ratios and provide numbers for products of the form $|U_{qq'}|f_P$, where $U_{qq'}$ is the corresponding KM matrix element, and f_P is the decay constant. For meson semileptonic decays the physical observable is the differential decay rate, $d\Gamma/dq^2$, which up to known factors, is proportional to $|U_{qq'}f_+(q^2)|^2$, where $f_+(q^2)$ is a complex form factor and q denotes the transferred momentum between initial and final mesons. In the last case the experimental teams usually provide numerical values for products of the form $|U_{qq'}f_+(0)|$, where $f_+(0)$ is the semileptonic decay form factor at zero-momentum transfer. It is clear that from such measurements one cannot find two unknowns, say $|U_{qq'}|$ and f_P . This can be done if and only if one can find independent constraints on KM matrix moduli; fortunately these ones are provided by unitarity. Thus the main aim of the paper is to show how the unitarity property of the KM matrix can be transformed into a powerful tool for the determination of both matrix moduli and form factors directly from experimental data.

The unitarity constraints are presented in Sec. II where they are implemented in a χ^2 -form that depends only on KM matrix moduli. In Sec. III we present the decay formulas for superallowed $0^+ \rightarrow 0^+$ nuclear and neutron β decays, and those for leptonic and semileptonic decays. They depend on KM matrix moduli and specific decay parameters such as decay constants, f_P , and form factors $|f(q^2)|$, etc, that are implemented in an other χ^2 -piece. In Sec. IV we cite the experimental papers from which we take data that are used in our fit, and in Sec. V we present numerical results. The paper ends by Conclusion.

II. UNITARITY CONSTRAINTS

The use of $|U_{ij}|$ as independent parameters raises an important problem. This means that we have to solve the consistency problem between moduli and unitarity property, which all amounts to obtaining the necessary and sufficient conditions on the set of numbers $|U_{ij}|$ to represent the moduli of an exact unitary matrix. After that we have to find a device for applying these conditions to experimental situation where data are known modulo uncertainties.

Both these problems were recently solved, and a procedure for recovering KM matrix elements from error affected data was provided in [9]. These unitarity constraints say that the four independent parameters $s_{ij} = \sin \theta_{ij}$ and $\cos \delta$ should take physical values, i.e. $s_{ij} \in (0, 1)$ and $\cos \delta \in (-1, 1)$, when they are computed via equations set:

$$\begin{aligned} V_{ud}^2 &= c_{12}^2 c_{13}^2, \quad V_{us}^2 = s_{12}^2 c_{13}^2, \quad V_{ub}^2 = s_{13}^2 \\ V_{cb}^2 &= s_{23}^2 c_{13}^2, \quad V_{tb}^2 = c_{13}^2 c_{23}^2, \\ V_{cd}^2 &= s_{12}^2 c_{23}^2 + s_{13}^2 s_{23}^2 c_{12}^2 + 2s_{12} s_{13} s_{23} c_{12} c_{23} \cos \delta, \\ V_{cs}^2 &= c_{12}^2 c_{23}^2 + s_{12}^2 s_{13}^2 s_{23}^2 - 2s_{12} s_{13} s_{23} c_{12} c_{23} \cos \delta, \\ V_{td}^2 &= s_{13}^2 c_{12}^2 c_{23}^2 + s_{12}^2 s_{23}^2 - 2s_{12} s_{13} s_{23} c_{12} c_{23} \cos \delta, \\ V_{ts}^2 &= s_{12}^2 s_{13}^2 c_{23}^2 + c_{12}^2 s_{23}^2 + 2s_{12} s_{13} s_{23} c_{12} c_{23} \cos \delta \end{aligned} \quad (1)$$

The above relations have been obtained by using the standard KM matrix form, [2], where $V_{ij} = |U_{ij}|$, and U_{ij} are KM matrix entries. In paper [9] it was shown that if the independent parameters are KM matrix moduli the reconstruction of a unitary matrix knowing its moduli is essentially unique. By consequence in the following the used independent parameters in all our phenomenological analyses will be V_{ij} moduli. Although only four of them can be independent the experimental data “force” us to use all the possible sets of four independent moduli, as it will be shown in the following, and a simple combinatorial evaluation shows that there are 57 such sets.

Relations (1) are repassing invariant, i.e. they have the same form after multiplication of all KM matrix rows and columns by arbitrary phases. More important is that they contain all unitarity constraints. It is easily seen that all the six relations such as

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1 \quad (2)$$

are a consequence of the above relations. The relations (2) are necessary conditions for unitarity fulfilment, but not *sufficient* as we show in the following. In fact relations (2) show that if they are satisfied then there exist a physical solution for s_{ij} . Indeed from any four independent moduli entering (1) one can get the mixing parameters s_{ij} and $\cos \delta$, i.e. the four independent parameters entering KM unitary matrix. For example, if $V_{us} = a$, $V_{ub} = b$, and $V_{cb} = c$ is one set of three independent moduli, from the first five equations (1) we find all the three mixing parameters

$$s_{13} = V_{ub} = b, \quad s_{12} = \frac{a}{\sqrt{1-b^2}}, \quad s_{23} = \frac{c}{\sqrt{1-b^2}} \quad (3)$$

The other parameter, δ , can be obtained from anyone of the last four equations. If we choose the sixth equation one gets

$$\cos \delta = \frac{(1-b^2)(V_{cd}^2(1-b^2) - a^2) + c^2(a^2 + b^2(a^2 + b^2 - 1))}{2abc\sqrt{1-a^2-b^2}\sqrt{1-b^2-c^2}} \quad (4)$$

and from the remaining relations three new $\cos \delta$ formulas similar to (4). The above relation shows that $\cos \delta$ is an other invariant in the Jarlskog sense depending of four independent moduli, and *CP*-violation phase can be measured via relations such as (4).

If we make use of the last four relations (1) we get only one solution for mixing parameters and $\cos \delta$. Thus depending on the chosen four independent moduli set the number of solutions varies between one and four. Because there are 57 such groups one get 165 different expressions for $\cos \delta$. They take the same numerical value when are computed via Eqs. (1), *if and only if all the six relations similar to Eq.(2) are exactly satisfied*. If the moduli matrix generated by four independent moduli is compatible with unitarity then $\cos \delta \in (-1, 1)$, and outside this interval when the corresponding matrix is not compatible. For example if we choose $V_{us} = 2257/10^4$, $V_{ub} = 359/10^5$, $V_{cd} = 2256/10^4$, and $V_{cb} = 415/10^4$, by using the necessary relations similar to (2), the corresponding moduli matrix is

$$|U| = \begin{pmatrix} \frac{\sqrt{9490466219}}{10^5} & \frac{2257}{10^4} & \frac{359}{10^5} \\ \frac{141}{625} & \frac{3\sqrt{10526471}}{10^4} & \frac{83}{2 \times 10^3} \\ \frac{\sqrt{580181}}{10^5} & \frac{\sqrt{10982}}{2500} & \frac{\sqrt{9982648619}}{10^5} \end{pmatrix} \quad (5)$$

and from (4) one gets $\cos \delta \approx 0.64088$, showing that the above moduli matrix, (5), comes from an exact unitary matrix.

If we modify the previous numerical V_{us} value by adding to it the small quantity 3×10^{-4} the mixing parameters are still physical, only s_{12} is modified by a very small quantity, and respectively all the square root entries of (5), necessary for fulfilment of all the six relations similar to (2). In this case one gets $\cos \delta \approx -1.42427$, which shows that the new moduli matrix, $|U|$, is not compatible with unitarity, even it *exactly* satisfies all the six relations (2).

If one computes the J invariant one finds in the above two cases

$$J^2 = 6.317 \times 10^{-10}, \text{ and } J^2 = -1.106 \times 10^{-9} \quad (6)$$

Thus the physical conditions for unitarity compatibility are $\cos \delta \in (-1, 1)$, and $J^2 > 0$, respectively, and from a theoretical point of view they are equivalent. For numerical computations the use of $\cos \delta$ formulas, like (4), seems to be more efficient because of their great sensitivity to small moduli variation.

The real physical cases are those where the central value moduli matrices, directly determined from data, or from a fit do not exactly satisfy relations (2), but only approximately; for example for a good fit the difference could be $10^{-5} - 10^{-7}$, i.e. rather small from a phenomenological point of view. In these cases the different formulas for $\cos \delta$ provide different values, physical and unphysical, even if the mixing parameters take physical values as in previous example. Hence physical reality obliges us to implement the unitarity constraints

$$\cos \delta_i \approx \cos \delta_j, \quad i \neq j, \text{ all } \cos \delta_i \in (-1, 1) \quad (7)$$

into a χ^2 fitting device, and our choice is

$$\chi_1^2 = \sum_{j=u,c,t} \left(\sum_{i=d,s,b} V_{ji}^2 - 1 \right)^2 + \sum_{j=d,s,b} \left(\sum_{i=u,c,t} V_{ij}^2 - 1 \right)^2 + \sum_{i < j} (\cos \delta^{(i)} - \cos \delta^{(j)})^2, \quad -1 \leq \cos \delta^{(i)} \leq 1 \quad (8)$$

that enforces all unitarity constraints.

III. DECAY FORMALISM

In this section we present the decay formalism such as it is used for the description of available experimental data, formalism that allows us to define a second piece of the χ^2 -function by taking into account as much as possible the physical information.

Information on V_{ud} come from two important sources, superallowed $0^+ \rightarrow 0^+$ nuclear β decay, and neutron β decay. Superallowed $0^+ \rightarrow 0^+$ β decay between $T = 1$ analog states depends uniquely on the vector part of the weak interaction and, according to the conserved vector current hypothesis, its experimental ft value should be related to the vector coupling constant, which is a fundamental constant and by consequence has the same value for all such transitions, see [10], [11], [12] and [13]. This means that the following relation should hold

$$ft = \frac{K}{2|G_V|^2 |M_F|^2} = \text{const} \quad (9)$$

where $K/(\hbar c)^6 = 2\pi^3 \hbar \ln 2/(m_e c^2)^5$, G_V is the vector coupling constant for semi-leptonic weak interactions, and M_F is the Fermi matrix element which in this case is equal to $\sqrt{2}$. The ft value that characterizes any β transition depends on the total transition energy Q_{EC} , the half-life, $t_{1/2}$, of the parent state, and the branching ratio for the particular studied transition, [11]. The above relation is only approximately satisfied by a restricted data set, and for this set one defines a “corrected” value $\mathcal{F}t \equiv ft(1 + \delta'_R)(1 + \delta_{NS} - \delta_C)$, where δ'_R and δ_{NS} comprise the transition-dependent part of the radiative correction, while δ_C depends on the details of nuclear structure. In the above formula we take $|G_V^2| = g_V^2 V_{ud}^2$, with $g_V = 1$, and write it as

$$\mathcal{F}t = \frac{K}{2V_{ud}^2(1 + \Delta_R^V)} \quad (10)$$

where Δ_R^V is the transition-independent part of the radiative corrections whose last estimation given in [11] is

$$\Delta_R^V = (2.361 \pm 0.038)\% \quad (11)$$

Similarly for neutron β decay we make use of formula

$$V_{ud}^2(1+3\lambda^2) = \frac{4908.7(1.9)s}{\tau_n} \quad (12)$$

see [14], where τ_n is the neutron mean life and $\lambda = g_A/g_V$. In our approach V_{ud} , Δ_R^V , and λ are free parameters to be found from fit.

In SM the purely leptonic decay of a P meson, $P \rightarrow l\bar{\nu}_l$, proceeds via annihilation of the quark pair to a charged lepton and neutrino through exchange of a virtual W boson, and the branching fraction, up to radiative corrections, has the form

$$\mathcal{B}(P \rightarrow l\bar{\nu}_l) = \frac{G_F^2 M_P m_l^2}{8\pi\hbar} \left(1 - \frac{m_l^2}{M_P^2}\right)^2 f_P^2 V_{qq'}^2 \tau_P \quad (13)$$

where G_F is the Fermi constant, M_P and m_l are the P meson and l lepton masses, respectively, f_P is the decay constant, $V_{qq'}$ is the modulus of the corresponding KM matrix element, and τ_P is P lifetime.

The next simple decays involving V_{ij} moduli are the semileptonic decays of heavy pseudoscalar mesons, H , into lighter ones, P , whose physical observable is the differential decay rate, written as

$$\frac{d\Gamma(H \rightarrow P\ell\nu_\ell)}{dq^2} = \frac{G_F^2 V_{qq'}^2}{192\pi^3 M_H^3} \lambda^{3/2}(q^2) |f_+(q^2)|^2 \quad (14)$$

where $q = p_H - p_P$ is the transferred momentum, and $f_+(q^2)$ is the global form factor which is a combination of the two form factors generated by the vector part of the weak current. When the leptons are electrons, or muons whose masses are low compared to mass difference $m_H - m_P$, $\lambda(q^2)$ is the usual triangle function

$$\lambda(q^2) = (M_H^2 + M_P^2 - q^2)^2 - 4M_H^2 M_P^2 \quad (15)$$

For the decay $\bar{B} \rightarrow D\ell\nu$ the experimenters make use of an other variable, namely $w = (M_B^2 + M_D^2 - q^2)/(2M_B M_D)$. When the τ lepton is involved the above formulas are a little bit more complicated, see [15].

Usually, till now, the experimenters provided numerical values for products of the form $V_{qq'} f_+(0)$, but we expect that data of the form $V_{qq'} |f_+(q^2)|$ will be available in the near future for all semileptonic decays.

The second χ^2 -component which takes into account the experimental data has the form

$$\chi_2^2 = \sum_i \left(\frac{d_i - \tilde{d}_i}{\sigma_i} \right)^2 \quad (16)$$

where d_i are the theoretical functions one wants to be found from fit, \tilde{d}_i are the numerical values that describe the corresponding experimental data, while σ_i is the uncertainty associated to \tilde{d}_i . For semileptonic decays d_i could be of the form $d_i = |f_+(q_i^2)| V_{kl}$. In the following our fitting χ^2 -function will be

$$\chi^2 = \chi_1^2 + \chi_2^2 \quad (17)$$

As it is easily seen V_{ij} moduli enter naturally in all formulas that describe leptonic and semileptonic decays being, in our opinion, a strong argument for their use as fit parameters.

IV. EXPERIMENTAL DATA

In our analysis we used superallowed $0^+ \rightarrow 0^+$ nuclear β decays from [10]-[13], and the neutron lifetime from four papers: [16], [17], [18], [19], for V_{ud} determination. We also used four values for the β -asymmetry parameter A_0 , from papers [20], [21], [22], [23], and one for the electron-antineutrino correlation coefficient a_0 , [24], for λ determination.

V_{us} modulus is involved in kaon and pion leptonic and semileptonic decays, but also in ratio V_{us}/V_{ud} , as for example in Marciano relation, [25], that we write as

$$\frac{V_{us}^2 f_K^2}{V_{ud}^2 f_\pi^2} (1 + C_r) = \frac{\mathcal{B}(K \rightarrow \mu\bar{\nu}_\mu(\gamma)) \tau_\pi m_\pi (1 - \frac{m_\mu^2}{m_\pi^2})^2}{\mathcal{B}(\pi \rightarrow \mu\bar{\nu}_\mu(\gamma)) \tau_K m_K (1 - \frac{m_\mu^2}{m_K^2})^2} \quad (18)$$

where C_r is a radiative correction stemming from both π and K hadronic structures.

The last numerical values for the product $f_+^{\pi K}(0)V_{us}$ are given by KLOE collaboration, [26], and by FlaviaNet Working Group, [27], and in fit we made use of all the (little) different $f_+(0)V_{us}$ values corresponding to the five channels. We also used results from [28], [29], [30], [31], [32], [33], [34], [35], [36], [37], [38], [39] that give only a “mean value” for the above product.

V_{ub} is the most poorly determined modulus although there is much experimental information coming from decays $B \rightarrow \pi l\nu$, see [41], [42], [43], [44], [46], [47], [48]. In this papers the experimenters have been confronted with the known difficulty, getting two distinct parameters, V_{ub} and $|f_+^{B\pi}(q^2)|$, from their product measured from experimental data. Thus they used the form factor lattice computations to obtain V_{ub} values depending on q^2 , see [41], [42], and [48]. For fit we found three measurements for $f_B V_{ub}$, [43], [45], [46], and three fenomenological determinations involving $f_+^{B\pi}(0)V_{ub}$, [47], [48], [49].

V_{cd} and V_{cs} moduli enter the leptonic decays $D \rightarrow l\nu$, [50], [51], [52], and $D_s^+ \rightarrow l\nu$, [53], [54], [55], [56], [57], respectively, as well as the semileptonic decays $D \rightarrow \pi l\nu$, and $D \rightarrow K l\nu$, [59], [60], [61]. In the last three papers one find $V_{cq}f_+(0)$ values, for $q = d$, or s , and for the first time values on

$$|f_+(q^2)|V_{cq} = \sqrt{\frac{d\Gamma}{dq^2} \frac{24\pi^3 p_{K,\pi}^3}{G_F^2}} \quad (19)$$

[61], giving the possibility of extracting form factors directly from data. The above semileptonic decays allow the measurement of the ratio $V_{cd}f_+^{D\pi}(0)/V_{cs}f_+^{DK}(0)$, see [60], [61], [62], [63], [64], [65], and give an independent determination of the ratio $f_+^{DK}(0)/f_+^{D\pi}(0)$, which was considered a new independent parameter.

Finally from semileptonic decays $\bar{B} \rightarrow Dl\nu$ and $\bar{B} \rightarrow D^*l\nu$, [66], [67], [68], [69], [70], [71], [72], [73], [74], [75], [76], [77], one find V_{cb} , $\mathcal{G}(1)$ and $\mathcal{F}(1)$ parameters.

V. NUMERICAL RESULTS

Data from the above cited papers were used to define χ_2^2 , the second component of full χ^2 , which has a parabolic form in V_{ij}^2 . The first component χ_1^2 , (8), that contain all unitarity constraints, has a parabolic part, and one that is highly nonlinear in all V_{ij} . Thus we had to test the stability of the expected physical values against the strong non-linearity implied by unitarity. Eventually the chosen method was to modify all the measured central values in the same sense, plus and minus, respectively, proportional to their corresponding uncertainties.

An important assumption included in our approach is that numerical V_{ij} values must be the same irrespective of the physical processes used to determine them. Accordingly the other parameters, such as decay constants, f_P , form factors, $f_+(0)$, λ , etc., that parametrize each given experiment, have been considered as independent parameters to be obtained from fit, by applying the usual technique to obtain their mean values and uncertainties.

The stability tests provided sets of different moduli matrices that have been used to obtain a mean value matrix and its corresponding error matrix. The mean and uncertainty matrices have been computed by embedding unitary matrices into the double stochastic matrix set, see [9].

The central values and uncertainties of data used in fit are those published in the above cited papers, and we combined the statistical and systematic uncertainties in quadrature when experimentalists provided both of them.

The numerical values obtained from fit for decay constants, f_π , f_K , f_B , f_D , f_{D_s} , semileptonic form factors $f_+(0)$, Δ_R^V , λ , as well as the ratios f_K/f_π and, $f_+^{DK}(0)/f_+^{D\pi}(0)$ are given in TABLE I. All of them are in the expected range although many have big uncertainties.

Our approach allows a “fine structure analysis” of all experiments measuring one definite quantity, such as Δ_R^V , or $f_+^{\pi K}(0)$.

For example KLOE collaboration data, [26], and FlaviaNet Working Group data, [27], on $f_+(0)V_{us}$ lead to

$$\begin{aligned} f_+^{\pi K}(0)_{KLOE} &= 950.38 \pm 5.56, \text{ and} \\ f_+^{\pi K}(0)_{Flavia} &= 955.06 \pm 4.31, \end{aligned} \quad (20)$$

respectively, whose central values are a little bit different, but compatible between them at 1σ , and all together provide

$$f_+^{\pi K}(0)_{KLOE+Flavia} = 952.72 \pm 5.30 \quad (21)$$

$f_+^{\pi K}(0)$ from Table I has a precision of 1%, and the above value obtained from ten measurements has a precision of 0.56%.

TABLE I: Numerical values for decay constants f_P and form factors $f_+(0)$ in MeV units, and Δ_R^V and λ

| Parameter | Central Value | Uncertainty | Parameter | Central Value | Uncertainty |
|-----------------|---------------|-------------|-------------------------------|---------------|-------------|
| f_π | 131.131 | 1.522 | $f_+^{D\pi}(0)$ | 653.2 | 19.1 |
| f_K | 154.97 | 2.17 | $f_+^{DK}(0)$ | 751.8 | 10.4 |
| f_K/f_π | 1.1818 | 0.0042 | $f_+^{DK}(0)/f_+^{D\pi}(0)^a$ | 1.171 | 0.049 |
| f_B | 222.8 | 25.0 | $\mathcal{F}(1)$ | 957.5 | 57.7 |
| f_D | 207.6 | 9.8 | $\mathcal{G}(1)$ | 1,125.3 | 40.7 |
| f_{D_s} | 271.0 | 18.0 | $\Delta_R^V\%$ | 2.373 | 0.096 |
| $f_+^{K\pi}(0)$ | 955.34 | 9.27 | λ | -1.2686 | 0.0057 |
| $f_+^{B\pi}(0)$ | 214.9 | 13.4 | C_r | 0.002 | 0.0001 |

^a Data on $V_{cd}f_+^{D\pi}/V_{cs}f_+^{DK}$ from papers [60]-[65],

TABLE II: Δ_R^V central values and uncertainties provided by fit when using data from Refs. [10]-[13]

| Ref. | [10] | [11] | [12] | [13] | [11] |
|----------------|-----------|------------|------------|------------|------------------------|
| $\Delta_R^V\%$ | 2.373(96) | 2.399(112) | 3.361(196) | 2.294(136) | 2.361(38) ^a |

^a The “constant” Δ_R^V and its uncertainty, from [11]

In this approach one could obtain information about lepton universality in leptonic decays. Because the corresponding f_M decay constant should be same for both decays $M \rightarrow \mu\nu$ and $M \rightarrow \tau\nu$, a big difference between them could show a possible violation. As an example we chose $M = D_s^+$ meson since lattice computations provided a number for this decay constant, $f_{D_s} = 241(3)$, see [78], with a very small error. Our results are

$$\begin{aligned} f_{D_s^+ \rightarrow \mu\nu} &= 265.0 \pm 14.0 \\ f_{D_s^+ \rightarrow \tau\nu} &= 276.0 \pm 20.0 \end{aligned} \quad (22)$$

which are consistent with lepton universality. The above two numbers together with that from TABLE I completely disagree with that provided by lattice computations, being far away from theoretical prediction at 8σ , 13σ , and 10σ , respectively, where, $\sigma = 3$, is lattice uncertainty. The experimental spreading is, $246.0 \leq f_{D_s^+} \leq 311.0$, and the minimal and maximal values correspond to the branching ratios obtained for $\mathcal{B}(D_s^+ \rightarrow \mu^+\nu_\mu) = (5.15 \pm 0.63 \pm 0.20 \pm 1.29) \times 10^{-3}$, see [58], and to the branching ratio $\mathcal{B}(D_s^+ \rightarrow \tau\nu) = (8.0 \pm 1.3 \pm 0.6)\%$, given by Eq.(6) in paper [57], respectively. On the other hand if one computes the difference between each one of the above three values and that provided by lattice computation, divided by the corresponding experimental error σ obtained from fit one finds the same value, 1.17, which shows again the consistency of experimental data. In our opinion the lattice number is much underestimated. In contradistinction their value for $f_K/f_\pi = 1.189(7)$ is not far from the fit value $f_K/f_\pi = 1.1818(42)$.

Another unexpected result concerns Δ_R^V constancy, usually assumed in all four papers [10]-[13], assumption that is not confirmed by our analysis. However our result $\Delta_R^V = 2.373 \pm 0.096$ obtained from data [10] is in good concordance with the value given by relation (11), see TABLE II, but our uncertainty is 2.5σ higher than the theoretical one. In this case Δ_R^V spreading that results from paper [10] is, $2.2027 \leq \Delta_R^V \leq 2.472$, which corresponds to 7.1σ where $\sigma = 0.038\%$ is the theoretical uncertainty. The extremal nuclei are ^{22}Mg and ^{54}Co , respectively. The spreading obtained from Savard *et al* data, [13], corresponds to 10.8σ , and the extremal nuclei are ^{74}Rb and ^{34}Cl . However the difference between the central results from [10] and [13] is 2σ , which suggests that there is still room for computation improvements. The mean values and the corresponding uncertainties for all data from papers [10]-[13] are given in TABLE II.

Our approach allows the form factors determination. The paper [61] provided for the first time results on $V_{cq}|f_+(q^2)|$, $q = d, s$, and in TABLE III are given our determinations. Their graphic form is shown in FIG. 1 at the end of paper.

The precision of f_P and $f_+(0)$ determinations varies from 1% for $f_+^{K\pi}(0)$ to 11% for f_B . More about variability of the above parameters could be learnt from KM moduli matrix. Our fit result for KM central moduli values is

$$V_c = \begin{pmatrix} 0.974022 & 0.226415 & 0.0042512 \\ 0.226253 & 0.973323 & 0.0381075 \\ 0.0095692 & 0.0371307 & 0.999265 \end{pmatrix} \quad (23)$$

TABLE III: $|f_+(q^2)|$ form factors from $D \rightarrow \pi l \nu$ and $D \rightarrow K l \nu$ decays

| Bin | $f_+^\pi(q^2) (\pi^0 e^+ \nu_e)$ | $f_+^\pi(q^2) (\pi^- e^+ \nu_e)$ | $f_+^K(q^2) (K^- e^+ \nu_e)$ | $f_+^K(q^2) (\bar{K}^0 e^+ \nu_e)$ |
|-----|----------------------------------|----------------------------------|------------------------------|------------------------------------|
| 1 | 0.6895 ± 0.0576 | 0.7072 ± 0.0355 | 0.7798 ± 0.0136 | 0.7808 ± 0.0208 |
| 2 | 0.7514 ± 0.0709 | 0.7735 ± 0.0400 | 0.8281 ± 0.0146 | 0.8096 ± 0.0239 |
| 3 | 0.8442 ± 0.0841 | 0.7956 ± 0.0488 | 0.8435 ± 0.0156 | 0.8466 ± 0.0259 |
| 4 | 0.8926 ± 0.0974 | 0.9812 ± 0.0532 | 0.9113 ± 0.0198 | 0.8856 ± 0.0289 |
| 5 | 1.1005 ± 0.115 | 1.017 ± 0.065 | 0.9945 ± 0.0229 | 0.9175 ± 0.0331 |
| 6 | 1.3083 ± 0.151 | 1.101 ± 0.084 | 1.007 ± 0.027 | 1.024 ± 0.039 |
| 7 | 1.5789 ± 0.221 | 1.635 ± 0.111 | 1.128 ± 0.033 | 1.149 ± 0.048 |
| 8 | - | 1.759 ± 0.235 | 1.212 ± 0.042 | 1.090 ± 0.062 |
| 9 | - | 2.024 ± 0.301 | 1.303 ± 0.066 | 1.233 ± 0.092 |
| 10 | - | - | 1.561 ± 0.165 | 1.476 ± 0.210 |

where the digit numbers are those suggested by Mathematica rounding, when working in double precision. Its associated uncertainty matrix is

$$\sigma_{V_c} = \begin{pmatrix} 1.1 \times 10^{-6} & 1.9 \times 10^{-5} & 2.1 \times 10^{-5} \\ 3.6 \times 10^{-5} & 2.7 \times 10^{-4} & 3.1 \times 10^{-4} \\ 3.5 \times 10^{-5} & 2.9 \times 10^{-4} & 3.9 \times 10^{-4} \end{pmatrix} \quad (24)$$

The last matrix has been obtained with the help of stability tests. One such matrix is (25) that was obtained when all the central measured values have been modified with plus one tenth from the corresponding uncertainty. Although such a modification is highly improbable from an experimental point of view, it brings to light the variation direction for all parameters entering the fit.

$$V_+ = \begin{pmatrix} 0.974021 & 0.226332 & 0.0074844 \\ 0.226114 & 0.973083 & 0.044512 \\ 0.0124317 & 0.043391 & 0.998891 \end{pmatrix} \quad (25)$$

For example the above three matrices show that V_{ud} is precisely determined with five digits, while V_{us} , V_{cd} , and V_{cs} only with three digits. V_+ matrix also shows the high V_{ub} volatility, such that future data could lead to higher values for it than that given by V_c matrix. In fact matrices V_+ and V_c show that $f_B \in (123.5, 222.8)$. Although our value for $V_{ub} = (4.25 \pm 0.02) \times 10^{-3}$ is higher than that from PDG fit, [2], it is compatible with that obtained in Ref. [48], $V_{ub} = (4.1 \pm 0.2_{st} \pm 0.2_{syst}^{+0.6}_{-0.4, FF}) \times 10^{-3}$.

The new data from D leptonic, [50]-[52], and semileptonic decays, [59]-[61], combined with the new data from $\bar{B} \rightarrow D l \nu$ and $\bar{B} \rightarrow D^* l \nu$ [66]-[70], changed the KM moduli values, in particular those from the last row and column, see [2], p. 150.

A big step forward will be the measurement of q^2 dependence for products of the form $|f_+(q^2)V_{qb}|$, where $q = u, c$, for $B \rightarrow \pi l \nu$, $\bar{B} \rightarrow D l \nu$ and $\bar{B} \rightarrow D^* l \nu$ decays, similar to that done for D semileptonic decays. The simplest case is that of $|f_+(q^2)V_{ub}|$ because there are measured data in twelve bins which can be transformed in values for the product $|f_+(q^2)V_{ub}|$, [48]. Such measurements will allow a more precise V_{ub} and V_{cb} determination, and, by consequence, it will provide better values for all KM matrix moduli from the first two rows, and, perhaps, the first hints for new physics beyond SM, if any.

V_c matrix provides numerical values for δ and angles of the standard unitarity triangle, as follows

$$\begin{aligned} \delta &= (89.96 \pm 0.36)^\circ, \quad \alpha = (64.59 \pm 0.27)^\circ, \\ \gamma &= (89.98 \pm 0.06)^\circ, \quad \beta = (25.49 \pm 0.28)^\circ \end{aligned} \quad (26)$$

The δ value could be interpreted as a maximal violation of CP symmetry because $\sin \delta \approx 1$. A surprising change is the shape of the standard unitarity triangle that is now a right triangle, since $\gamma \approx 90^\circ$. The Jarlskog invariant is

$$J = (3.567 \pm 0.007) \times 10^{-5} \quad (27)$$

Similar results are obtained from V_+ matrix even if V_{ub} is almost twice bigger than that from V_c ; they are

$$\begin{aligned} \delta &= (90.0 \pm 0.2)^\circ, \quad \alpha = (54.1 \pm 0.1)^\circ, \\ \gamma &= (90.0 \pm 0.4)^\circ, \quad \beta = (36.0 \pm 0.2)^\circ \end{aligned} \quad (28)$$

The above results show that δ and γ angles are independent of V_{ub} variation, while α and β are moderately dependent. Thus experimental results on the product $|f_+(q^2)V_{ub}|$ could lead to the determination of all the angles of the standard unitarity triangle.

The small angles uncertainties show that V_c and V_+ matrices are highly compatible with unitarity constraints and all $165 \cos \delta$ formulas provide very close each other values for all of them.

A comparison with lattice computations from [79] shows that $f_+^{D\pi}(0)$ and $f_+^{DK}(0)$, as well as their ratio are in good agreement with the experimental values obtained by us, and our uncertainties are smaller. An open problem remains lattice computations of f_B and $f_+^{B\pi}(0)$ parameters.

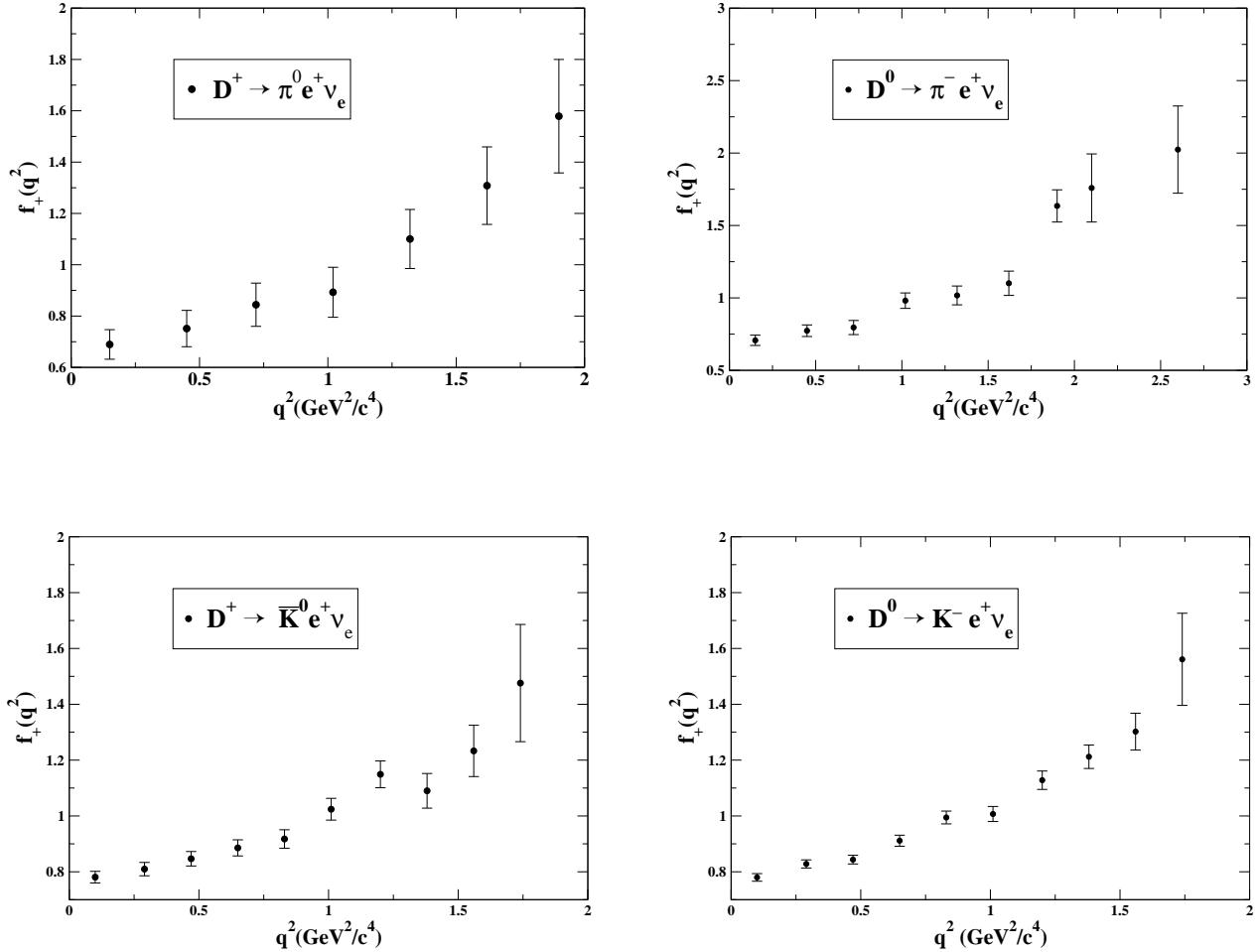


FIG. 1 $|f_+(q^2)|$ Form Factors from $D \rightarrow \pi l \nu$ and $D \rightarrow K l \nu$ decays

VI. CONCLUSION

In this paper we presented a phenomenological tool that allows determination of KM moduli, semileptonic form factors and decay constants directly from experimental data. It is based on a repartitioning invariant implementation of unitarity constraints that makes use of KM matrix moduli as fit parameters. These constraints are strong enough and give a consistent picture of nowadays flavor physics, and until now provide no signals for new physics beyond the SM. However there is a discrepancy between theoretical $f_{D_s^+}$ lattice computation value for the $D_s^+ \rightarrow l \nu$ decay constant, and the experimental value obtained from fit, that suggests that the numerical value is highly underestimated.

A feature of our tool is that all measurable parameters are each other strongly correlated, a little modification of one of them propagates to all the other parameters, property which is a consequence of unitarity constraints.

The new data from D leptonic and semileptonic decays, and those from $\bar{B} \rightarrow D(D^*)l \nu$ led to a significant change of KM moduli and of standard unitarity triangle shape. A crucial step forward would be the measurement of $B \rightarrow \pi l \nu$

form factors in bins of 0.5 GeV^2 that will allow a better V_{ub} determination, and a stabilization of moduli values entering the first two rows.

Taking into account that our approach gives reliable results for many parameters entering flavor physics an important task comes to the lattice community to improve their numerical algorithms, because detection of new physics could be the outcome of both experimentalists and theorists.

Acknowledgements. We acknowledge a partial support from ANCS Contract No 15EU/06.04.2009.

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